WESTMINSTER SCHOOL

THE CHALLENGE 2018

MATHEMATICS II

Tuesday 1st May 2018

Time allowed: 1 hour 30 minutes

You will need a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

Please write in black or blue ink.

1 Alex had £51·35 when he set out to the shops. He paid for seven muffins at £1·19 each and eleven giant cookies. When he got home, he had £27·84 left. How much was each cookie?

2 Sam ran 800 metres in 2 minutes and 28 seconds. Tom ran 1500 metres in 4 minutes and 37 seconds. Which of them ran faster on average? Show the working you do to decide.

3 **a** **i** What is the result of adding $\frac{1}{2}(x+1)$ and $x+\frac{1}{2}$?

ii By what would you need to multiply $\frac{10b}{a}$ to make $\frac{5a}{b}$?

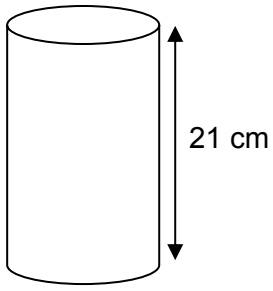
b Make V the subject of

$$P=\frac{RT}{V-b}.$$

c Solve the equation

$$\frac{x+10}{3}-2(x-1)=3.$$

4 The diagram shows a cylinder with a height of 21 cm and a volume of 1336 cm^3. What is the surface area of the cylinder?



5 **a** The volume of a meteorological balloon increases by 14% for each 1000 metres it rises through the atmosphere. The volume of the balloon is 5488 cm^3 at a height of 3000 metres. What was its volume at ground level?

b The pressure in a meteorological balloon decreases by the same percentage, P, for each 1000 metres it rises through the atmosphere. The pressure in the balloon is 101300 Pascals at ground level and 34800 Pascals at a height of 8000 metres.

What is the pressure in the balloon at a height of 5000 metres?

[The Pascal is a unit of pressure equal to one Newton per square metre].

6 There are 2018 students in a school. Every student must study either History or Geography, but students can study both subjects. The head teacher knows that between 80% and 85% of the students study History, and that between 30% and 40% study Geography.

a What are the maximum and minimum numbers of students who study Geography?

b What are the maximum and minimum numbers of students who study both subjects?

7 The tuck shop sells sandwiches, which all cost the same amount.

David buys eight sandwiches, and pays between £13 and £14 for them.

a If a sandwich costs K pence, what is the range of possible values of K?

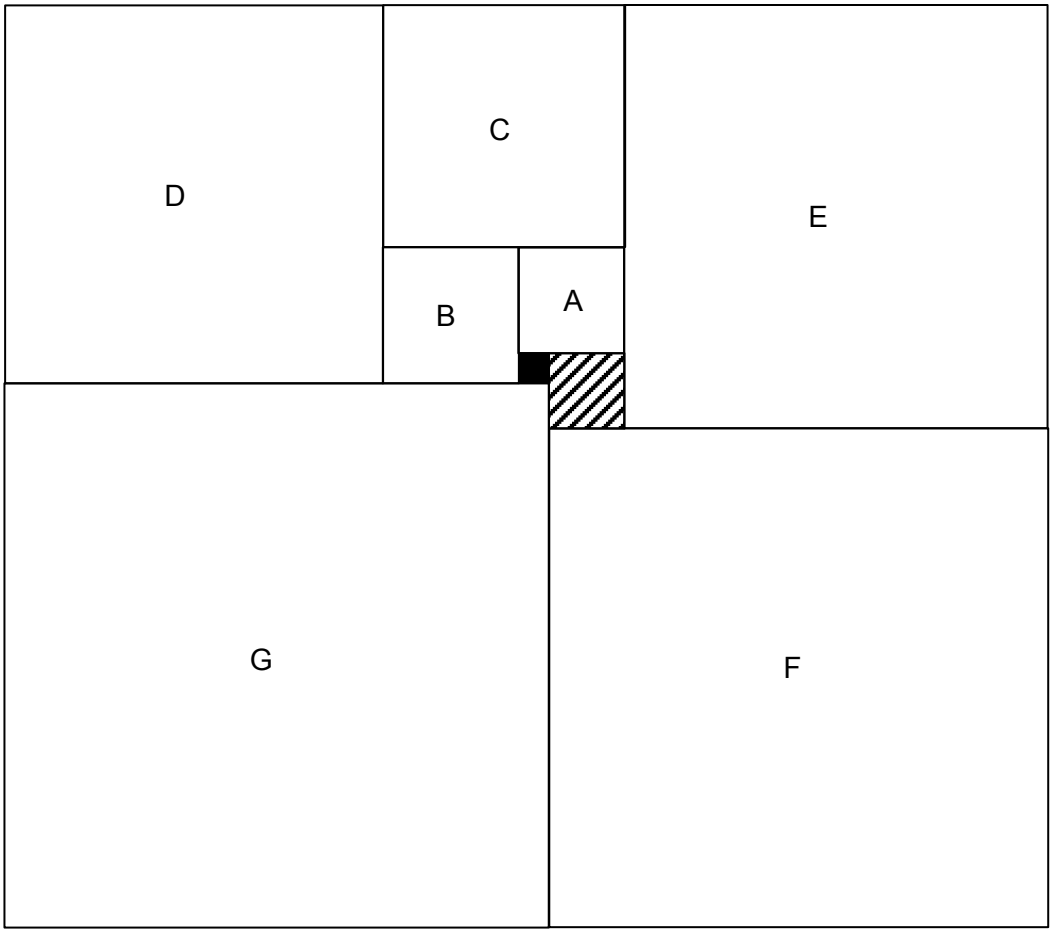
Ezra buys eleven sandwiches, and pays between £17 and £18 for them.

b Find the cost of a sandwich.

8 The diagram shows how nine squares have been fitted together to form a rectangle.
The smallest square (black) has side length 2 cm and the second smallest square (shaded) has side length x cm.

a Find the side lengths of the other squares, labelled A to G, in terms of x.

b Find the value of x, and hence the length and width of the rectangle.



9 A Subonacci list of numbers is formed as follows:

- the first two numbers in the list are given
- each subsequent number is one less than the sum of the previous two.

For example, a Subonacci list could start as follows

2 6 7 12

because $7 = 2 + 6 - 1$ and $12 = 6 + 7 - 1$.

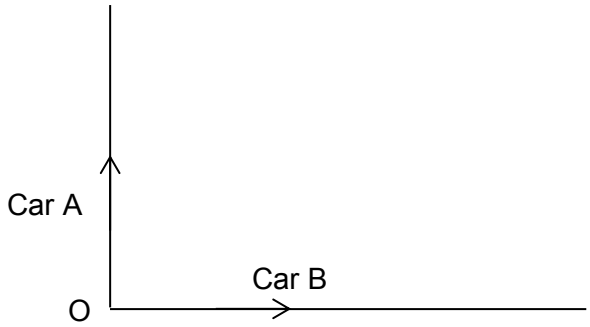
The first number in a Subonacci list is 4 and the second is 7.

a Find the next four numbers in the list.

b Is the 2018^{th} number in the list even or odd? Justify your answer.

c Prove that none of the numbers in the list is a multiple of three.

10 The diagram shows two straight roads, one heading due North and one heading due East from point O.



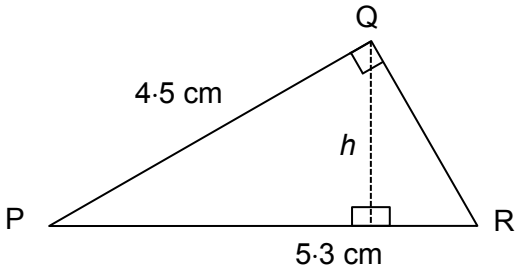
Car A heads North and Car B East; they leave O simultaneously.
Car A travels at 27·4 metres per second, and Car B travels at 30·8 metres per second.

a How long does it take before Car A and Car B are 1000 metres apart?

When they are 1000 metres apart, Car A and Car B turn and head directly towards each other, at the same speeds as before.

b How long is it before they collide?

11 **a** The diagram shows triangle PQR. Angle PQR is a right angle



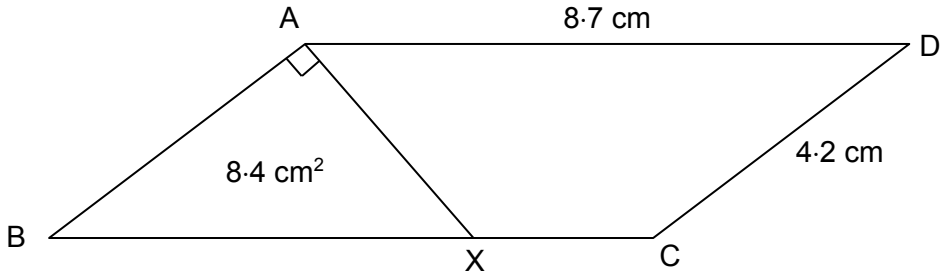
PR = 5·3 cm and PQ = 4·5 cm.

i Find QR.

ii Find the area of triangle PQR.

iii Find height *h*, correct to three significant figures.

b The diagram shows a parallelogram ABCD. Angle BAX is a right angle.



AD = 8·7 cm and CD = 4·2 cm.
The area of triangle ABX is 8·4 cm^2.
Find the area of trapezium AXCD.

12 The point A has co-ordinates (7, 2). The point O has co-ordinates (0,0)

B is a reflection of A in the line $y = x$.

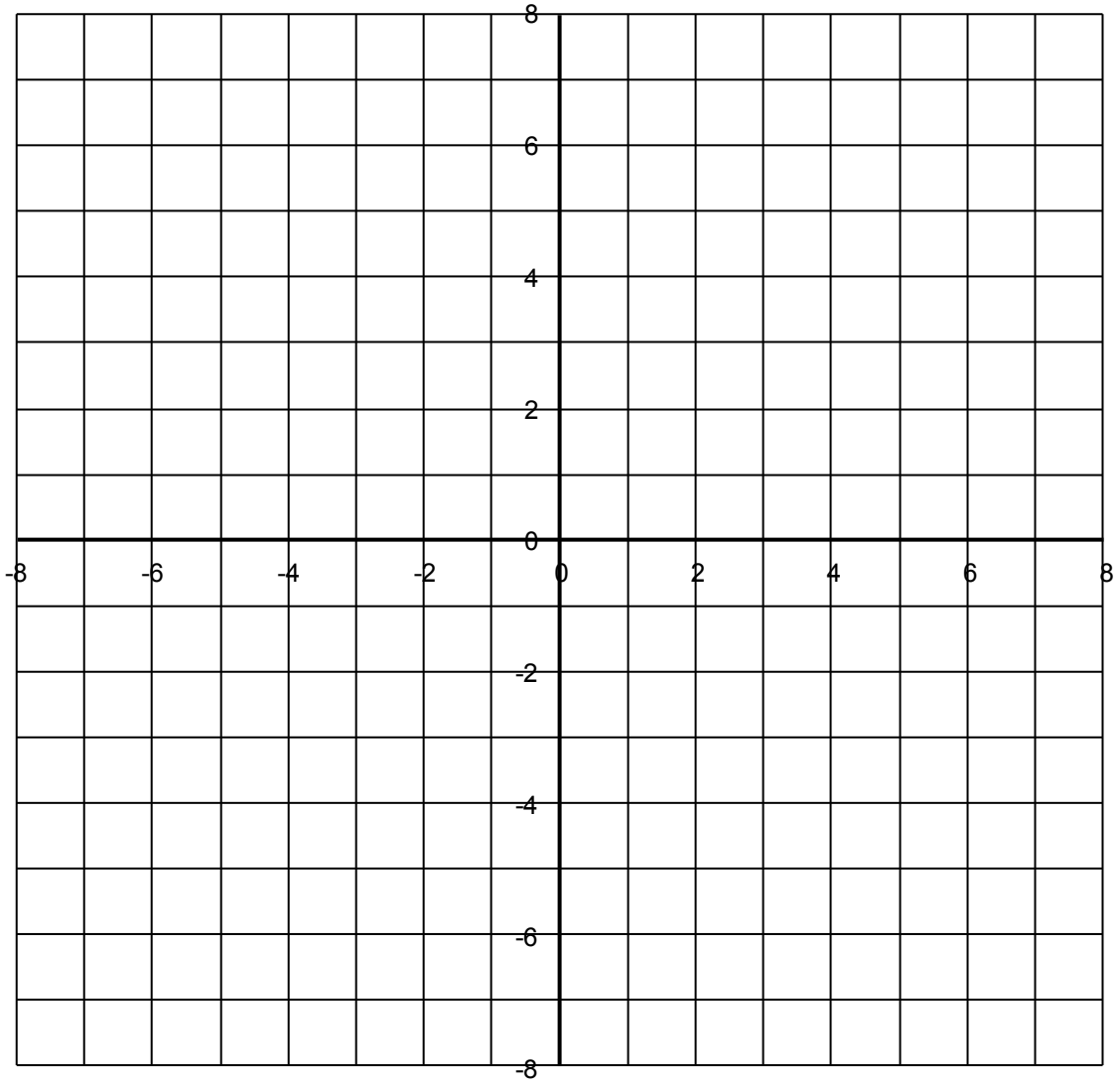
C is a reflection of A in the y axis.

D is a rotation of A by 90° clockwise about O.

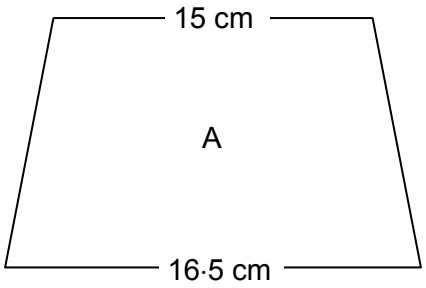
a Work out the co-ordinates of points B, C and D.

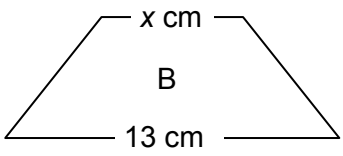
b Find the area of quadrilateral ABCD.

Use the grid below to help you if you wish, but do all your working in the answer booklet.



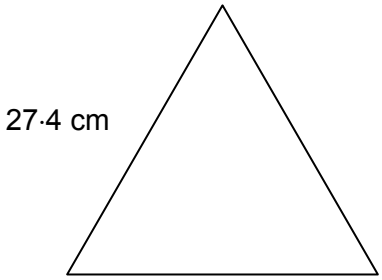
13 The diagram shows trapezium A and trapezium B.



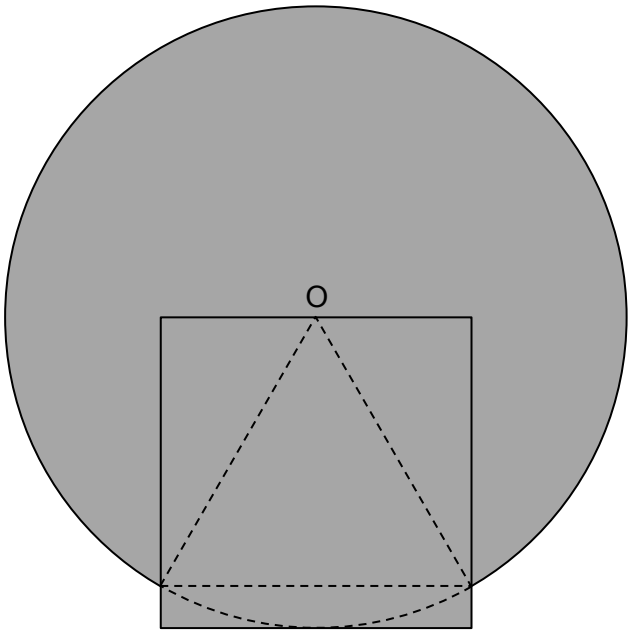


The height of trapezium A is twice that of trapezium B.
The area of trapezium A is three times that of trapezium B.
Find x.

14 **a** Find the area of the equilateral triangle shown below.



The circle below has radius R cm. Point O is the centre of the circle and the midpoint of one of the sides of a square of side length R cm.



b Find the total shaded area if $R = 27{\cdot}4$.

c Find R if the total shaded area is 1445 cm^2.